



# Making Learners (More) Monotone

Tom Viering, Alexander Mey, Marco Loog

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UNIVERSITY OF  
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TU Delft

Code available:

<https://github.com/tomviering/monotone>

# More Data

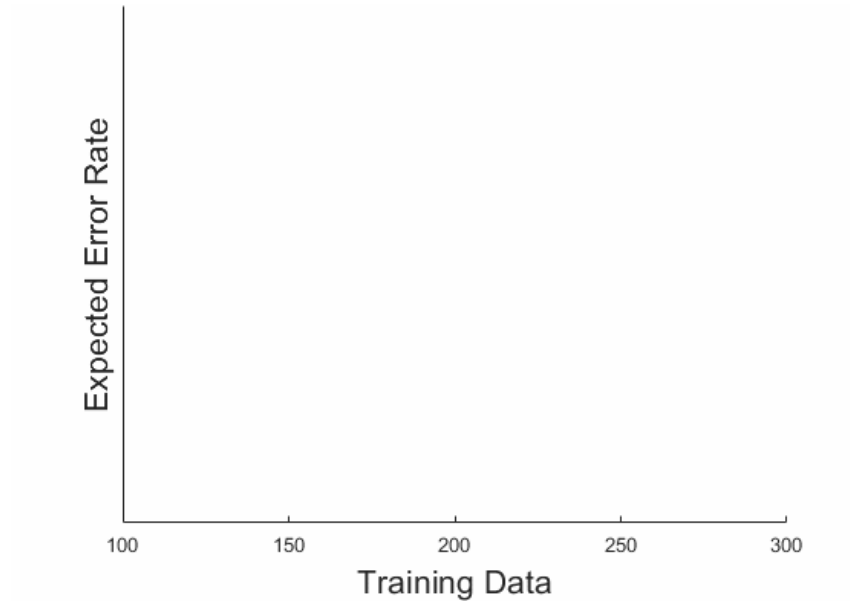
=

# Better Model

More Data  
=  
Better Model

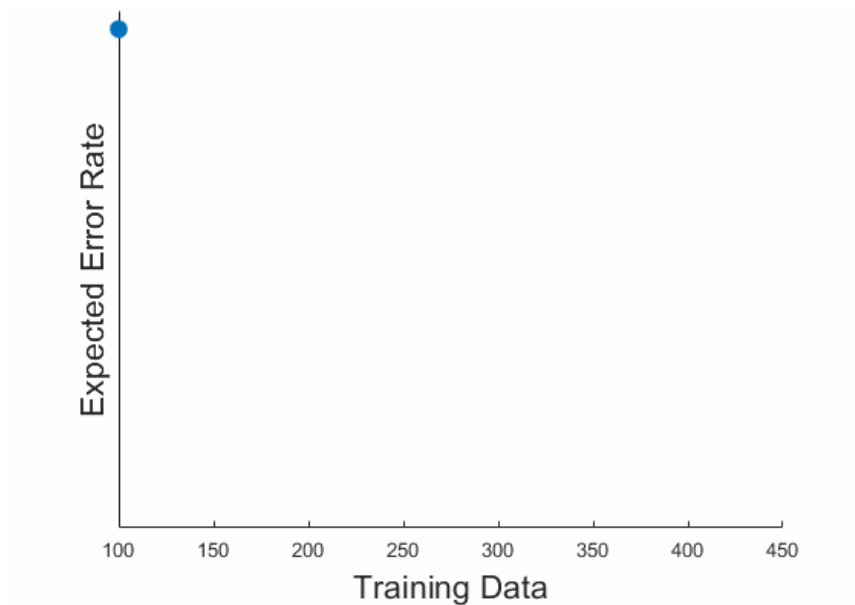
[Oppen 1990], Peaking [Duin, 1995], Dipping [Loog 2012],  
Double Descent [Belkin 2019], Deep Double Descent [Nakkiran 2019], Monotonicity  
of Learning [Viering 2019], Risk Monotonicity [Loog 2019], [Loog 2020]

# Expected Learning Curve



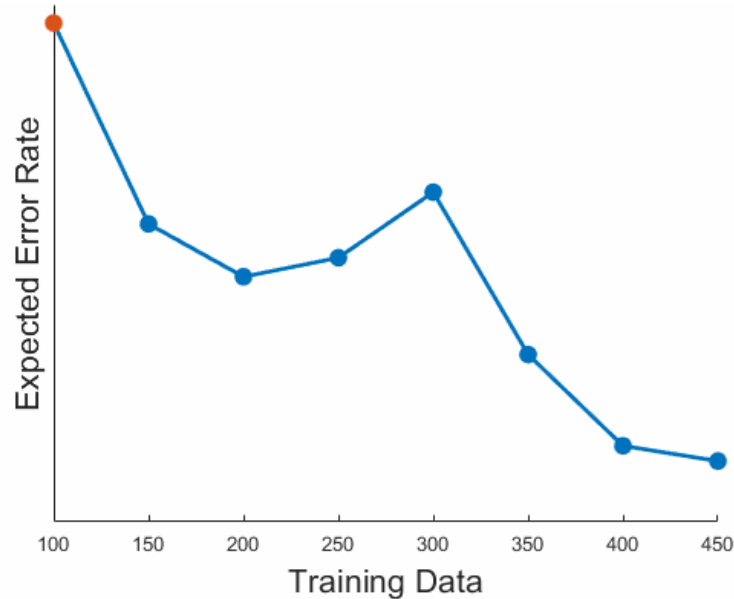
Expected = Averaged over multiple training datasets

# Expected Learning Curve



Peaking Dataset [Duin, 1995]

# What we want

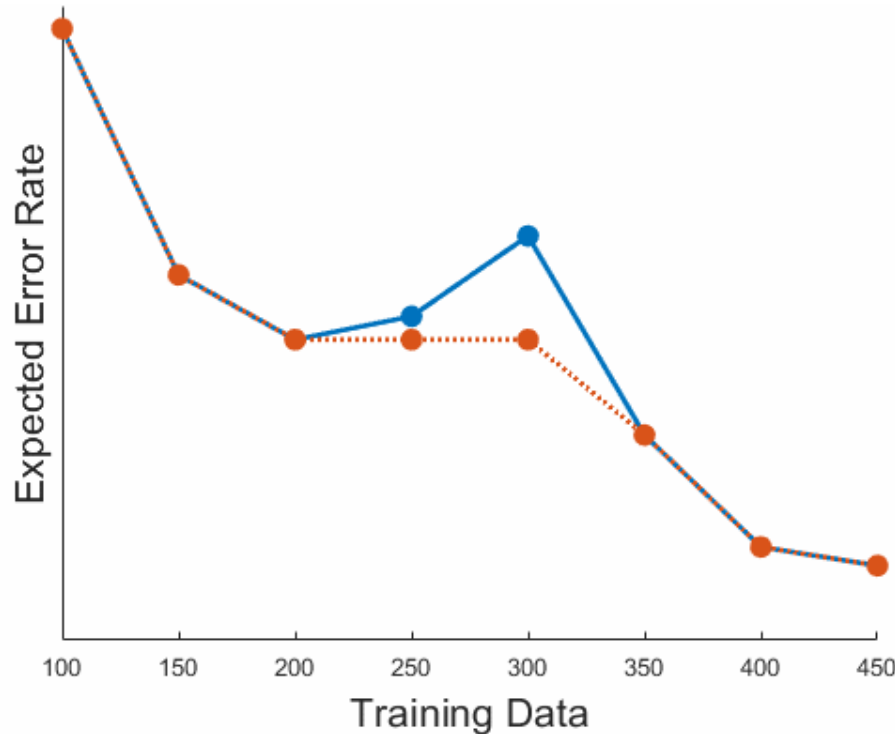


**Wrapper Algorithm:** makes learning curve of any classification model monotone

# Wrapper Algorithm

- Two ingredients
  - Model selection
  - Conservativeness

# Idea 1: model selection



Pseudocode SIMPLE

For each round

Get new data

Train new model

If new model better

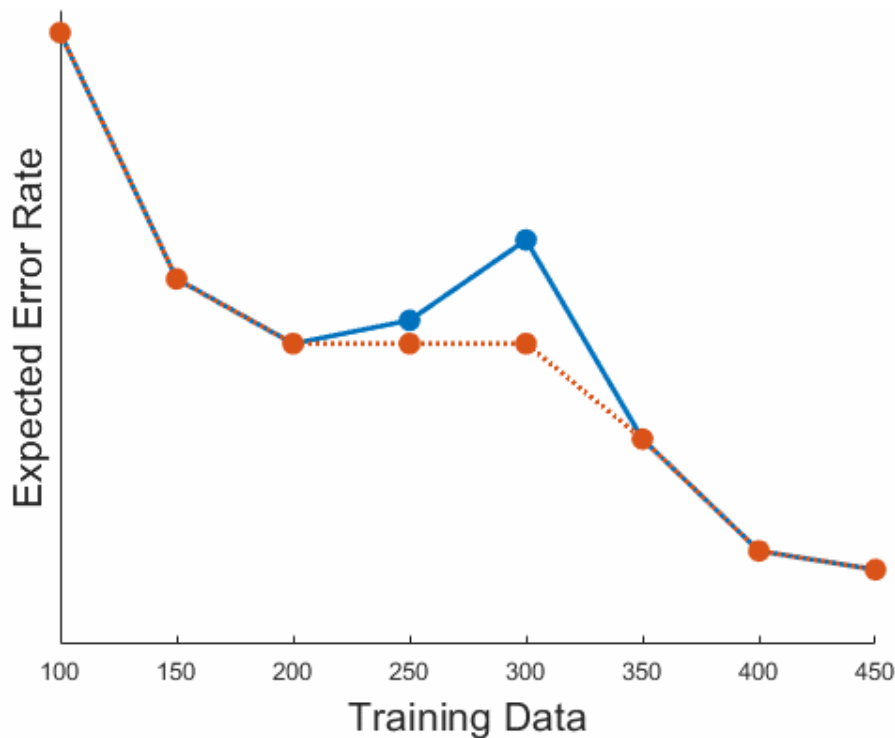
Use new model

Else

Use previous best



# Idea 1: model selection



Pseudocode SIMPLE

For each round

Get new data

Train new model

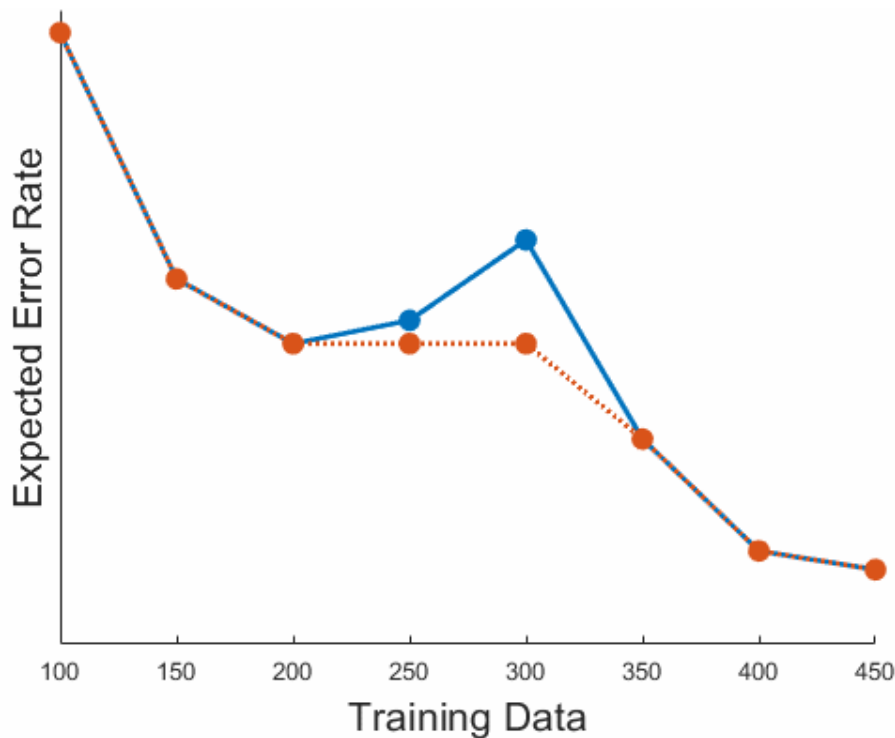
**If new model better**

Use new model

Else

Use previous best

# Idea 1: model selection



Pseudocode SIMPLE

For each round

Get new data

Split in val, train

Train new model

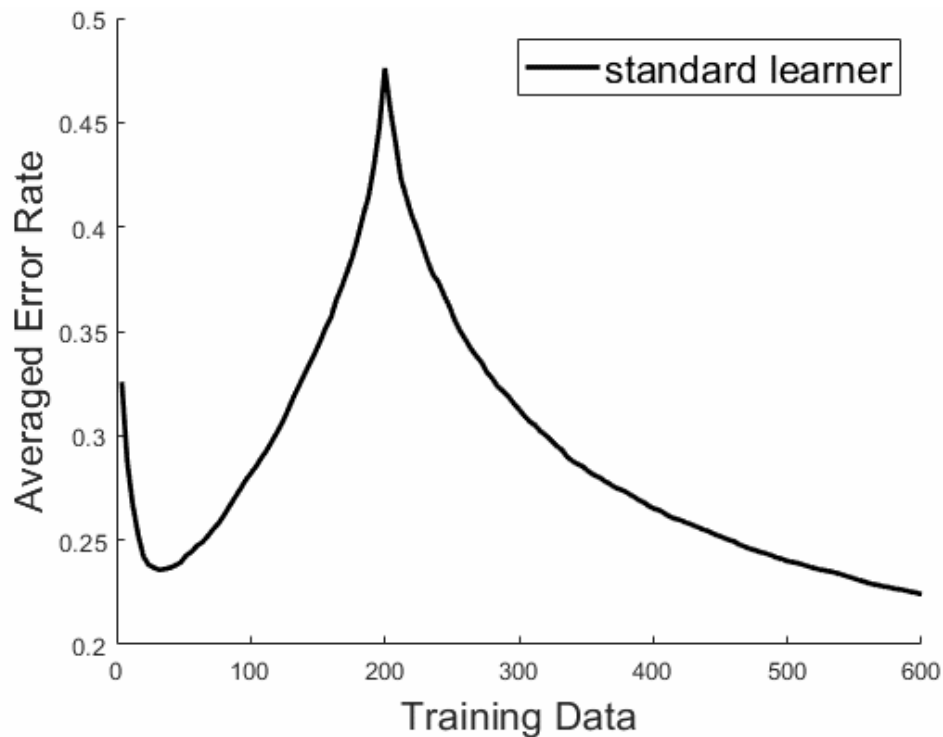
If new better on val

Use new model

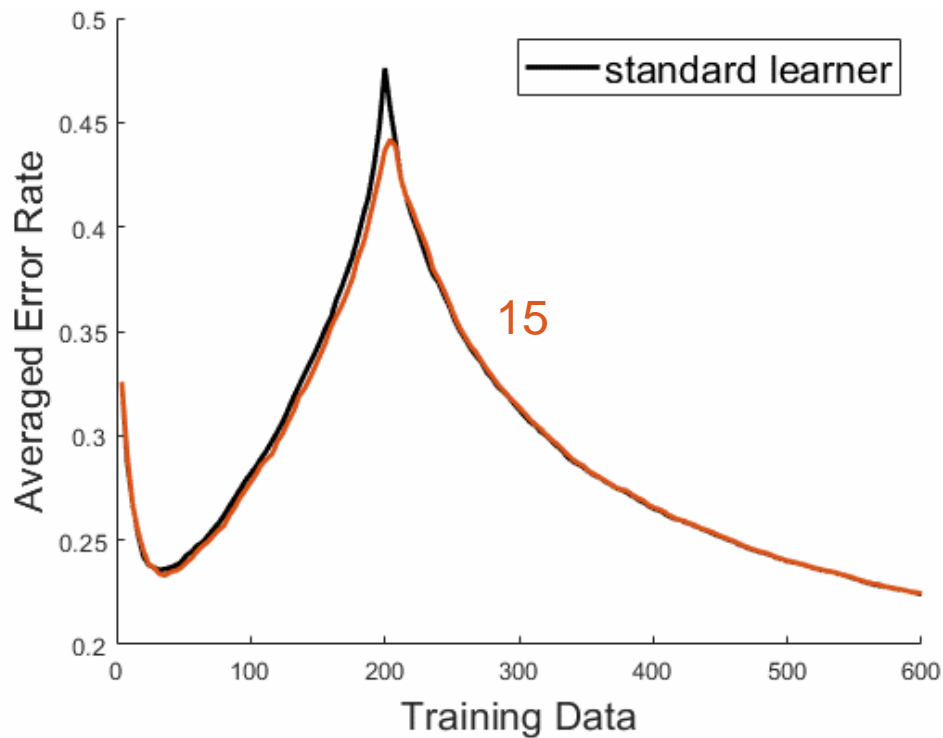
Else

Use previous best

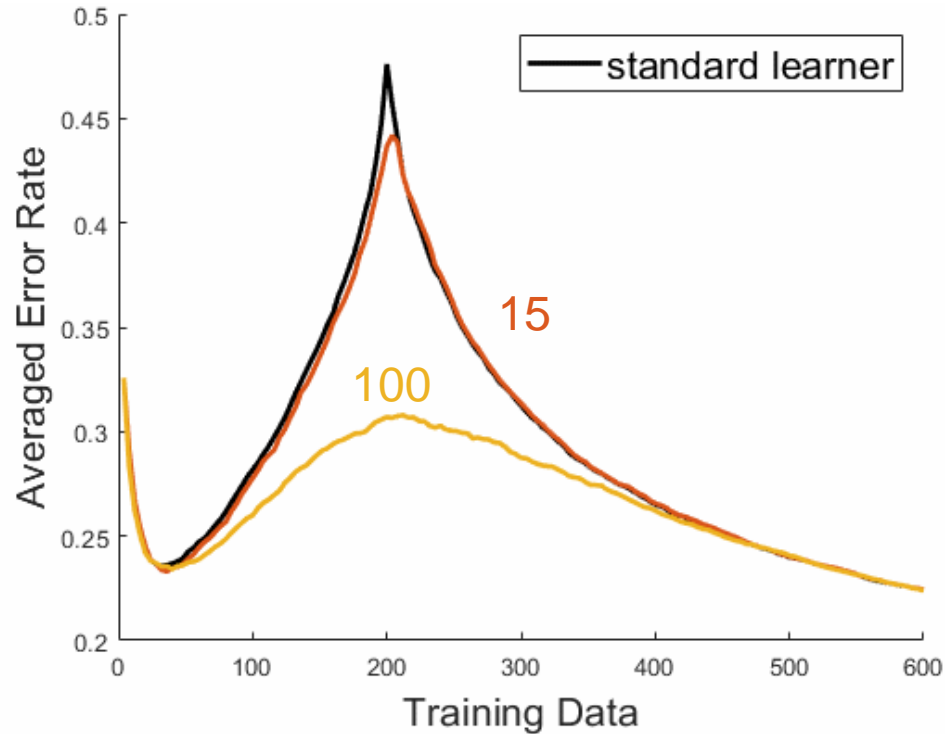
# Is SIMPLE good enough?



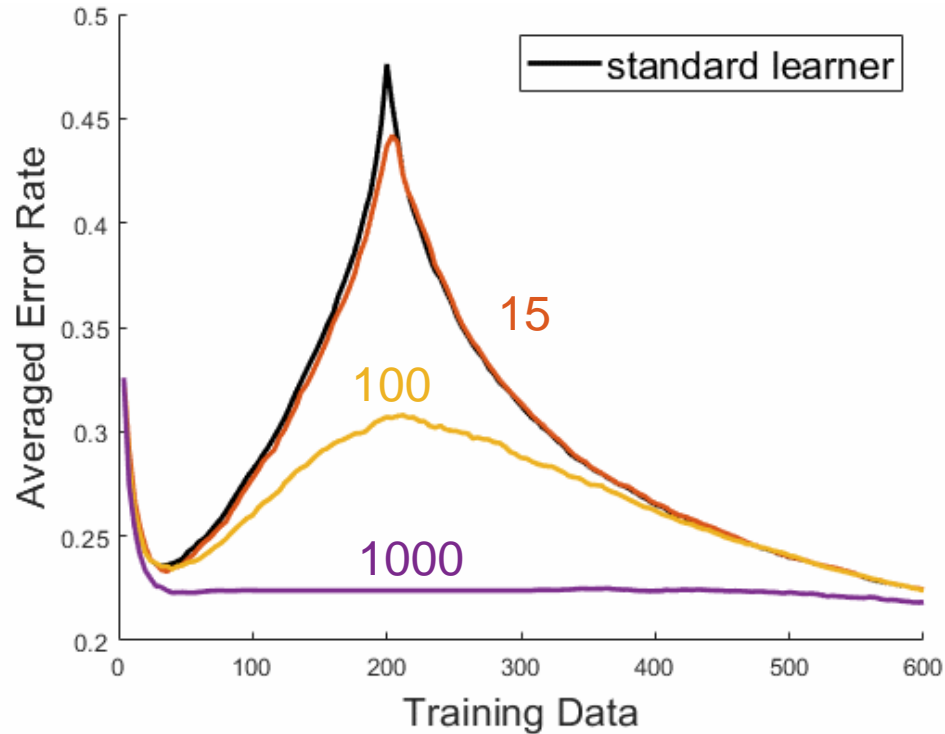
# Is SIMPLE good enough?



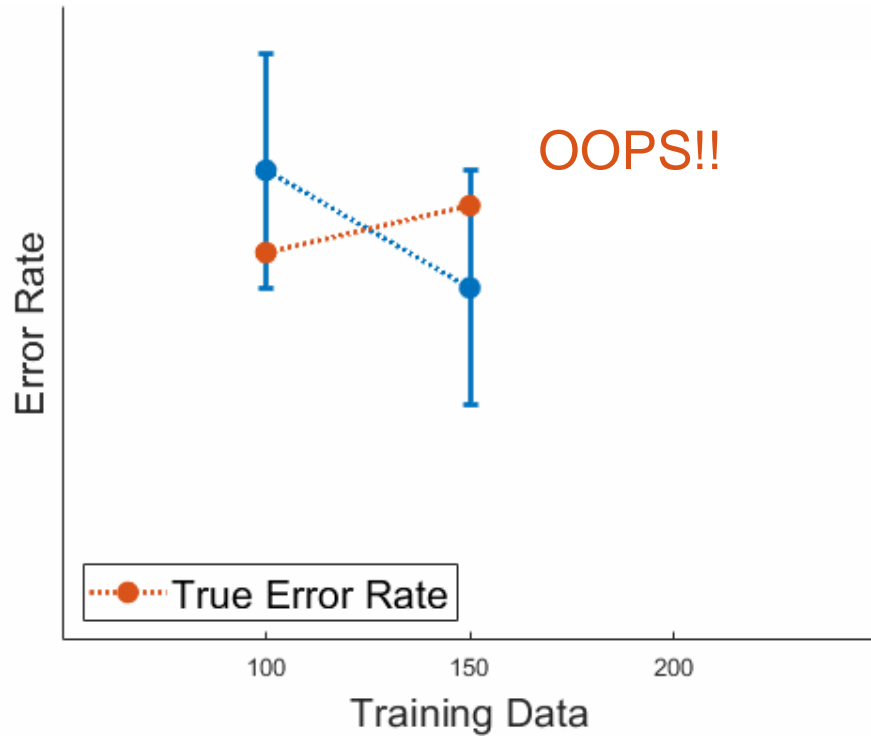
# Is SIMPLE good enough?



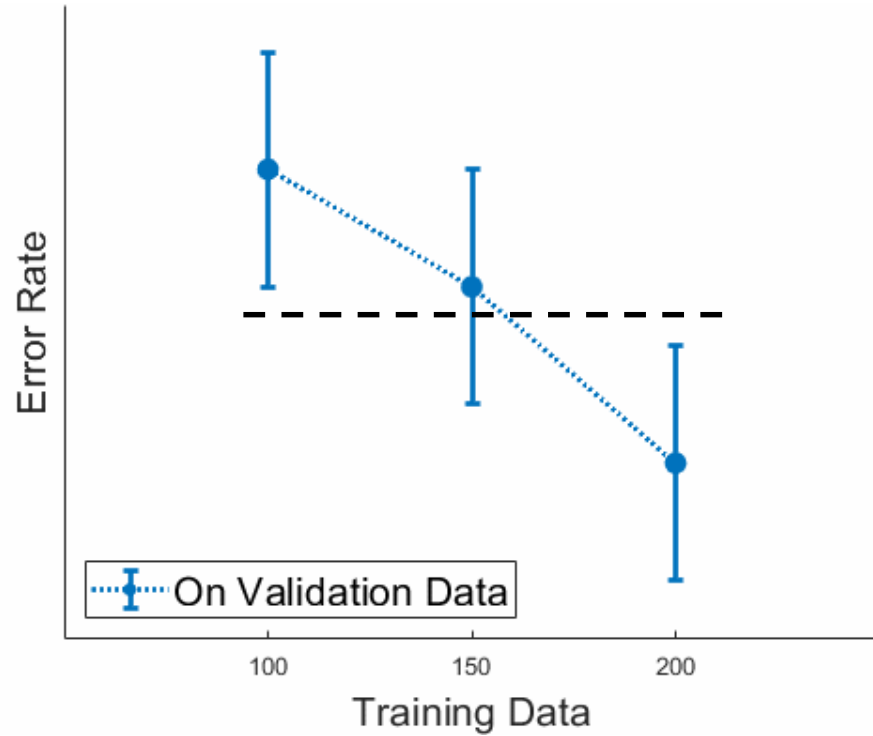
# Is SIMPLE good enough?











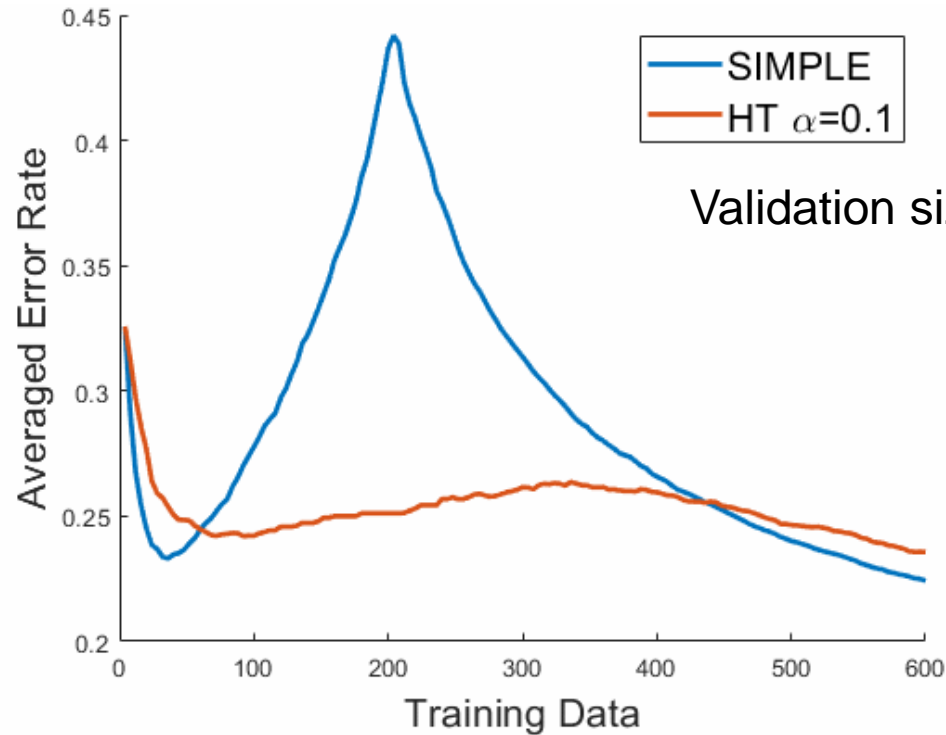
# Idea 2: Conservativeness

- Hypothesis test = conservative
  - Only switch to worse model with probability  $< \alpha$
- Significance level  $\alpha \in (0, \frac{1}{2}]$ 
  - Lower  $\alpha$  = more conservative

# Theoretical Guarantees for HT

1. With probability  $(1 - \alpha)^n$  a single learning curve is monotone
  - Key assumption: i.i.d. data
  - Doesn't say anything about expected learning curve
2. Wrapper algorithm is consistent
  - Under some conditions...

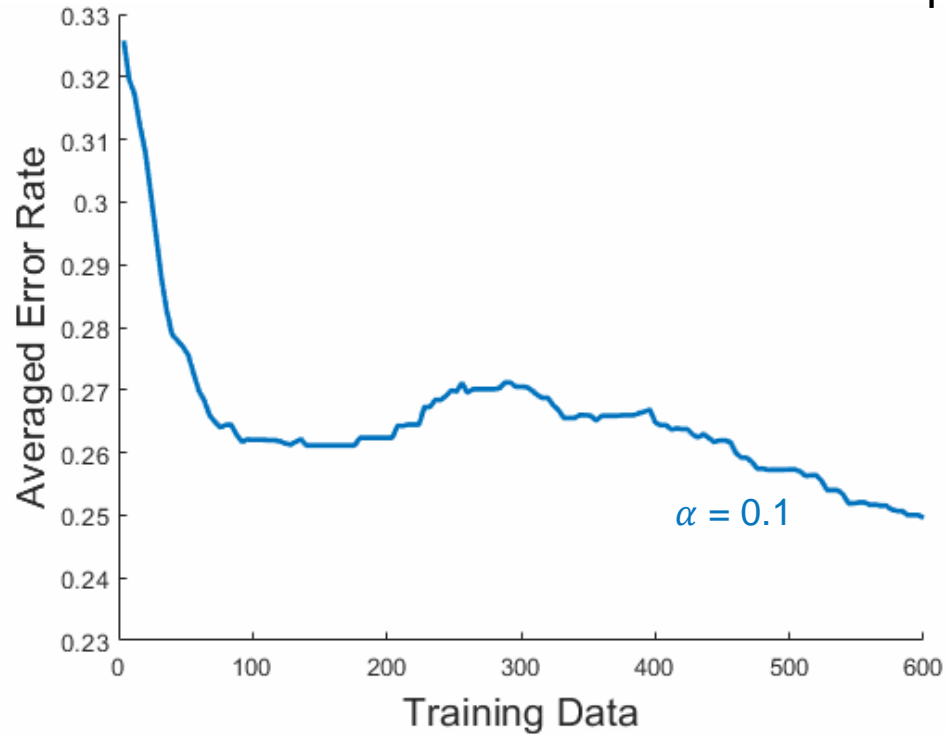
# Empirical Results



Validation size the same (15)

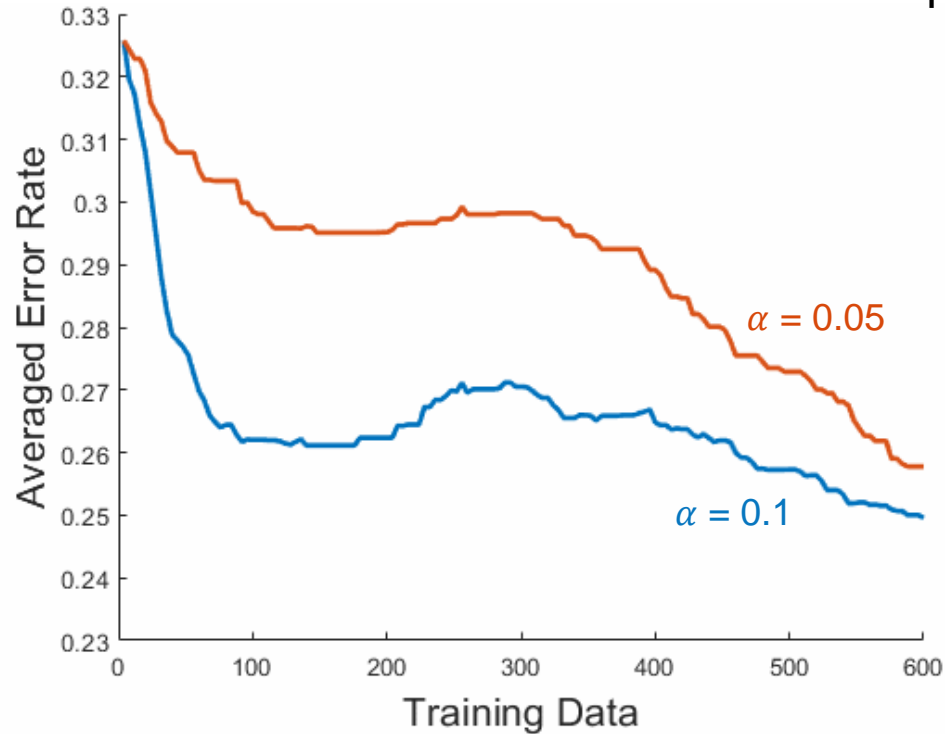
# Tuning $\alpha$

Very small validation set of 5 samples



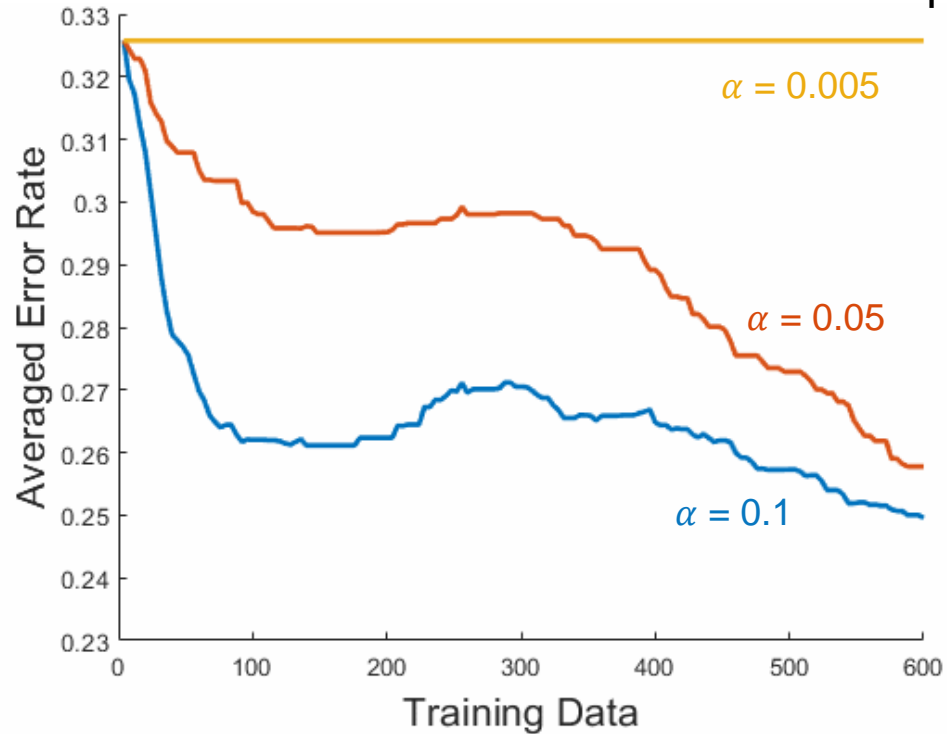
# Tuning $\alpha$

Very small validation set of 5 samples



# Tuning $\alpha$

Very small validation set of 5 samples



# Benchmark

- On Peaking, Dipping, MNIST
- Several baselines
  
- HT is by far the most monotone
- HT is competitive in performance, but learns slightly slower
- More monotone than guaranteed



# Discussion

- Parameter  $\alpha$
- Expected curve monotone?

# Conclusion

- Make any model monotone with high probability!
- Key ingredients to achieve monotonicity
  - Model selection
  - Conservativeness

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References for non-monotone behavior:

[Duin, 1995] Small sample size generalization ('peaking dataset')

[Loog 2012] The dipping phenomenon

[Belkin 2019] Reconciling modern machine-learning practice and the classical bias variance trade-off

[Nakkiran 2019] Deep Double Descent: Where Bigger Models and More Data Hurt

[Viering 2019] Open problem: Monotonicity of learning.

[Loog 2019] Minimizers of the Empirical Risk and Risk Monotonicity

[Loog 2020] A Brief Prehistory of Double Descent

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